Power Factor in Electrical Power Systems with Non-Linear Loads

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Abstract.
Traditional methods of Power Factor Correction typically focus on displacement power factor and therefore do not achieve the total energy savings available in facilities having both linear and non–linear loads. Only through Total Power Factor Correction can the savings and power quality be maximized. This paper provides a simplified explanation of both Power Factor (PF) and Total Power Factor (TPF) for various electrical systems having both linear and non–linear loads. Electrical diagrams and waveforms are combined with representative methods for analysis and help to demonstrate the application of mathematical calculations.

The concept of Total Power Factor is frequently misunderstood yet it is an important consideration for electrical installations which employ power electronics equipment. An understanding of Total Power Factor, along with the potential problems, is an important consideration when applying non linear loads such as VFD's, DC motor drives, Uninterruptible Power Supplies, and other power or frequency converters including SCR controllers.

The article explains the difference between Power Factor and Total Power Factor, and the influence of non–linear, harmonic producing loads on overall power system quality, reliability and energy efficiency.

Power factor with linear loads

When the loads connected to the system are linear and the voltage is sinusoidal, the power factor is calculated with the following equation:

$$pf = \cos(\varphi) \tag{1}$$

Unfortunately, this formula has led to a misunderstanding of the power factor concept. Power factor is the proportional relation of the active power (or working power) to the apparent power (total power delivered by the utility or consumed by the load). Using this definition, the power factor must be calculated as:

$$pf = \frac{P}{S} \tag{2}$$

When the loads are linear and the voltage is sinusoidal, the active, reactive and apparent power are calculated with the following equations:

$$P = VI\cos(\varphi) \tag{3}$$

$$Q = VI \sin(\varphi) \quad (4)$$

$$S = VI \quad (5)$$

and we can easily see that the power factor is calculated according to the equation (1), which is the resultant of the vector relationship between the active, reactive and apparent power shown in figure 1.

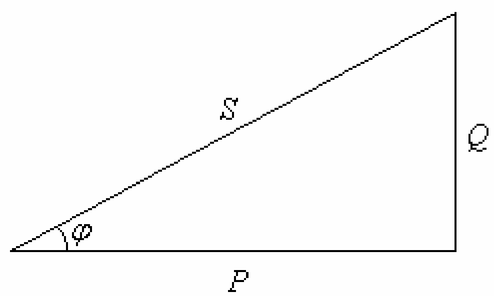

Figure 1

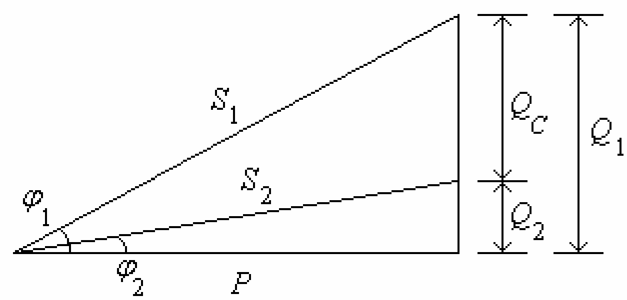

Figure 2

A low power factor means a that a low amount of the total power delivered or consumed (S) is used as working power (P) and a considerable amount is reactive power (Q). the purpose of power factor correction is to reduce the reactive component of the total power. This achieves a more efficient use of the energy because when the power factor is improved the working power is equal (or nearly equal) to the total power, and reactive power is zero or negligible. The most common way to correct power factor is by adding a capacitor bank connected in parallel with the power system. The capacitor bank (Q_C) supplies most of the reactive power needed by the load and a small amount is supplied by the utility (Q_2), as shown in figure 2. The original angle (φ_1) between the apparent and active power is reduced to a smaller value (φ_2) and the power factor is improved because $\cos(\varphi_2) > \cos(\varphi_1)$.

It is very important to note that the reduction in the angle obtained by the power factor improvement is a result of the vector relationship between the active, reactive and apparent power, but what we are really doing is reducing the reactive power, consequently the apparent power is also reduced and the power factor is increased.

Power factor with non–linear loads and sinusoidal voltage

When the loads are non–linear but the voltage is sinusoidal, the current has harmonics and the active, reactive and apparent power should not be calculated using the traditional methods as demonstrated by equations (3), (4) and (5). This means that the equation (1) can not be used to calculate the power factor when non–linear loads are concerned.

The active power is the mean (or average) value of the instantaneous power, so it can be calculated as:

$$P = VI_1 \cos(\varphi_1) \tag{6}$$

The rms value of current is a function of the total harmonic current distortion and the rms value of the fundamental component of current:

$$I = I_1 \sqrt{1 + THD_I^2} \tag{7}$$

The power factor, for non–linear loads, can be calculated using equations (5), (6) and (7):

$$pf = \cos(\varphi_1) \frac{1}{\sqrt{1 + THD_I^2}} \tag{8}$$

There are two terms involved in the calculation of the power factor: $\cos(\varphi_1)$ and $1/\sqrt{1+THD_I^2}$. The term $\cos(\varphi_1)$ is called *displacement power factor* (pf_{disp}) because it depends of the phase angle between the voltage and the fundamental component of the current, and it is similar to the power factor calculated with linear loads and sinusoidal voltage. The term $1/\sqrt{1+THD_I^2}$ is called *distortion power factor* (pf_{dist}) because it depends of the current harmonic distortion.

The power factor calculated as the product of the displacement power factor and the distortion power factor is known as *Total Power Factor* (pf_T):

Total Power Factor (pf_T), where $pf_T = fp_{disp} fp_{dist}$ (9)

If the reactive power of the loads increases, the displacement angle between the voltage and the fundamental component of the current also increases and the total power factor decreases. Likewise, if the total harmonic current distortion increases, the total power factor decreases. One can see by equation (8) that Total Power Factor will always be lower than the displacement power factor whenever harmonic distortion is present.

Total power factor correction can only be achieved when both displacement power factor and distortion power factor are corrected. This requires a two step process:

1. Reduce the displacement angle between voltage and current.
2. Reduce the total harmonic current distortion.

If either of these steps is taken without the other the total power factor will be increased but it may not be high enough to reach the minimum value required by the utility. Additionally, if one step is taken without the other, the Total Power Factor Correction and the corresponding efficiencies will not be achieved.

A vector relationship, as shown in figure 3, can be obtained from the active power (*P*), the fundamental reactive power (Q_{1F}) and the fundamental apparent power (S_{1F}) prior to displacement power factor improvement. This relationship allows us to visualize the effect that a capacitor bank $\left(Q_{C_F}\right)$ has on correcting the displacement power factor (figure 4). Whenever capacitors are used, care should be taken to avoid a resonant condition between the capacitor bank and the main transformer.

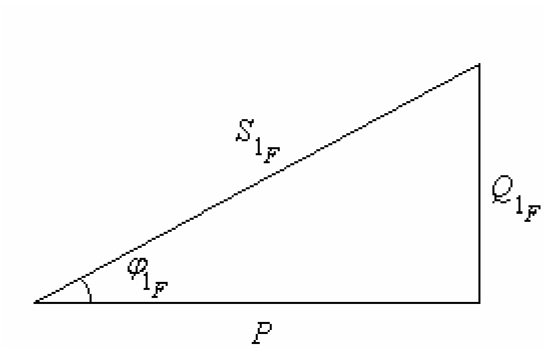

Figure 3

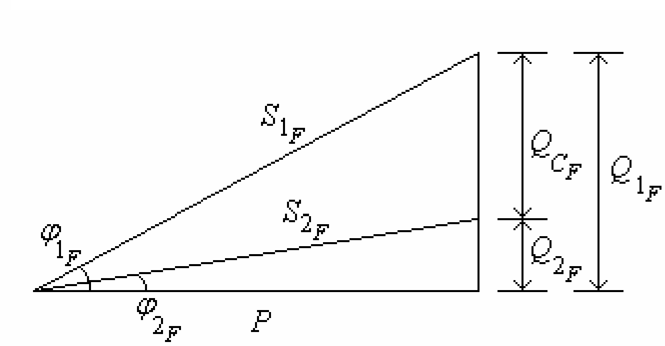

Figure 4

The *total power factor* can be improved by decreasing the harmonic current distortion, which is accomplished by using a filter instead of a capacitor bank. The capacitive part of the filter improves the displacement power factor, while the combination of the reactor and the capacitor bank decrease the total harmonic distortion of the current. A twofold result is achieved, that is improvement of the distortion power factor and improvement of the displacement power factor.

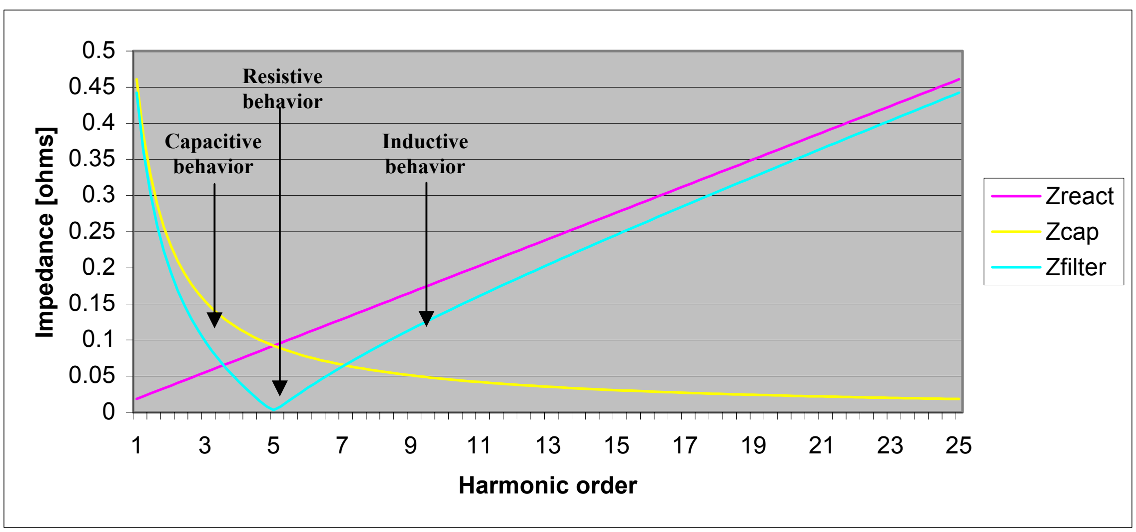

Figure 5

The Figure 5 shows the behavior of a harmonic passive filter as well as the behavior of its inductive and capacitive parts. Below the tuning harmonic the behavior of the filter is like a capacitor and above the tuning harmonic the behavior is like an inductor. At the tuning harmonic the behavior of the filter is like a resistor.

We can see that at the fundamental frequency, the filter acts like a capacitor bank because its reactance is basically capacitive, so the filter improves the displacement power factor.

At the tuning harmonic the filter is a very low impedance and a great amount of current at the tuning harmonic flows through it, decreasing the total harmonic current distortion and improving the distortion power factor. The improvement of both power factors (displacement and distortion) improves the total power factor.

If a capacitor bank were used instead of a filter, the displacement power factor would have been improved. If there is no resonance, the distortion power factor does not change and the total power factor increases only because the displacement power factor also increases, but the total power factor may not be high enough to reach the minimum value required by the utility. If a resonant condition is created between the capacitor bank and the main transformer, the total harmonic current distortion increases so the distortion power factor degrades and the final result is a low total power factor even with a capacitor bank and a high displacement power factor.

The figure 6 shows the behavior of the total power factor for different values of displacement power factor and total harmonic current distortion.

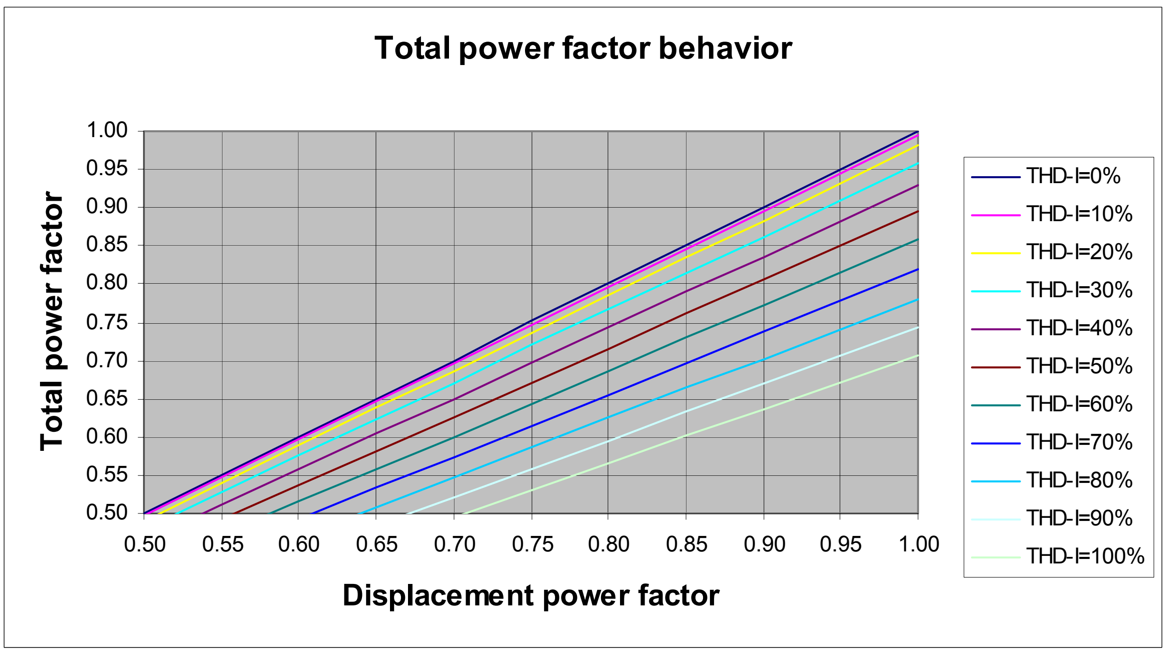

Figure 6

Power factor with non–linear loads and voltage distortion

When the loads are non–linear and the voltage is distorted the active, reactive and apparent power can not be calculated using traditional methods such as the equations (3), (4) and (5). The common equation (1) can not be used to calculate the power factor either.

The active power is the mean (or average) value of the instantaneous power. If the phase angles of the voltage harmonics are neglected, the active power can be calculated as:

$$P = \sum_{n=1}^{N} V_n I_n \cos(\varphi_n) \tag{10}$$

Now, the power factor can be calculated using equation (2):

$$pf = \frac{\sum_{n=1}^{N} V_n I_n \cos(\varphi_n)}{VI} \tag{11}$$

but, the voltage rms value is a function of the total harmonic voltage distortion and the rms value of the fundamental component of voltage:

$$V = V_1 \sqrt{1 + THD_V^2} \tag{12}$$

Using equations (7), (12) and (11), the power factor can be calculated as follows:

$$pf = \frac{P}{S_1} \times \frac{1}{\sqrt{1 + THD_I^2}\sqrt{1 + THD_V^2}} \tag{13}$$

There are two terms involved in the calculation of the power factor. The term P/S_1 is the relationship between the total active power (including harmonics) and the apparent fundamental power. This term should not be called displacement power factor because it involves the active power caused by the fundamental components and harmonics. The term $1/\left(\sqrt{1 + THD_I^2}\sqrt{1 + THD_V^2}\right)$ is the distortion power factor (pf_{dist}), which depends on the distortion of voltage and current. The power factor calculated as the product of the distortion power factor and the proportion of the total active power to the fundamental apparent power is the *total power factor* (pf_T):

$$pf_T = \frac{P}{S_1} pf_{dist} \tag{14}$$

The term P/S_1 can be expressed as:

$$\frac{P}{S_1} = \frac{V_1 I_1 \cos(\varphi_1)}{S_1} + \frac{\sum_{n=2}^{N} V_n I_n \cos(\varphi_n)}{S_1} \tag{15}$$

where $V_1 I_1 \cos(\varphi_1)/S_1$ is the displacement power factor (pf_{disp}), so the total power factor can be calculated as follows:

$$pf_T = \left(pf_{disp} + \frac{\sum_{n=2}^{N} V_n I_n \cos(\varphi_n)}{S_1} \right) pf_{dist} \tag{16}$$

In a similar way to the case of the non–linear loads and sinusoidal voltage, if the reactive power of the loads increases, the displacement angle between the fundamental components of voltage and current also increases and the total power factor decreases. If the distortion of current and voltage increases the distortion power factor decreases and the total power factor decreases as well.

If we want to improve the power factor then it is necessary to:

1. Reduce the displacement angle between the fundamental components of voltage and current.
2. Reduce the total harmonic distortion of both the current and the voltage.

A vector relationship, shown in figure 7, can be obtained from the fundamental active (P_1), reactive (Q_{1_F}) and apparent power (S_{1_F}) before the displacement power factor improvement. This vector relationship allows for the use of a capacitor bank (Q_{C_F}) to correct the displacement power factor (shown in figure 8), but again, a resonant condition between the capacitor bank and the main transformer must be avoided.

In a similar way to the case of power factor with non–linear loads and sinusoidal voltage, the total power factor can be improved by decreasing the harmonic current distortion, and this can be achieved by using a harmonic filter. The capacitive part of the filter improves the displacement power factor while the combination of the reactor and the capacitor bank decreases the total harmonic distortion of the current, thus improving the distortion power factor, as well as the total power factor.

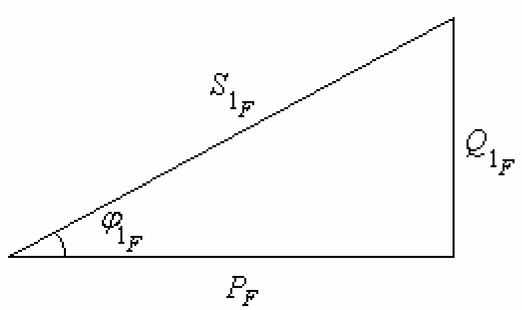

Figure 7

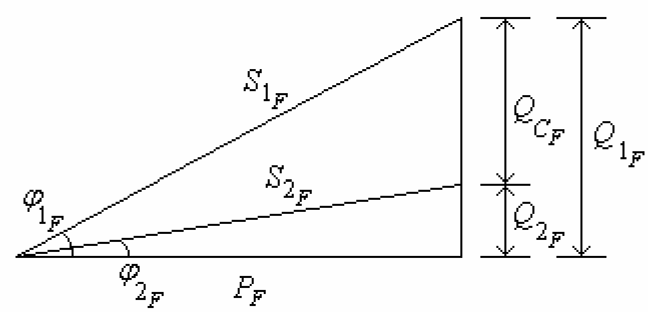

Figure 8

The figure 9 shows the behavior of the distortion power factor for different values of total harmonic current and voltage distortion.

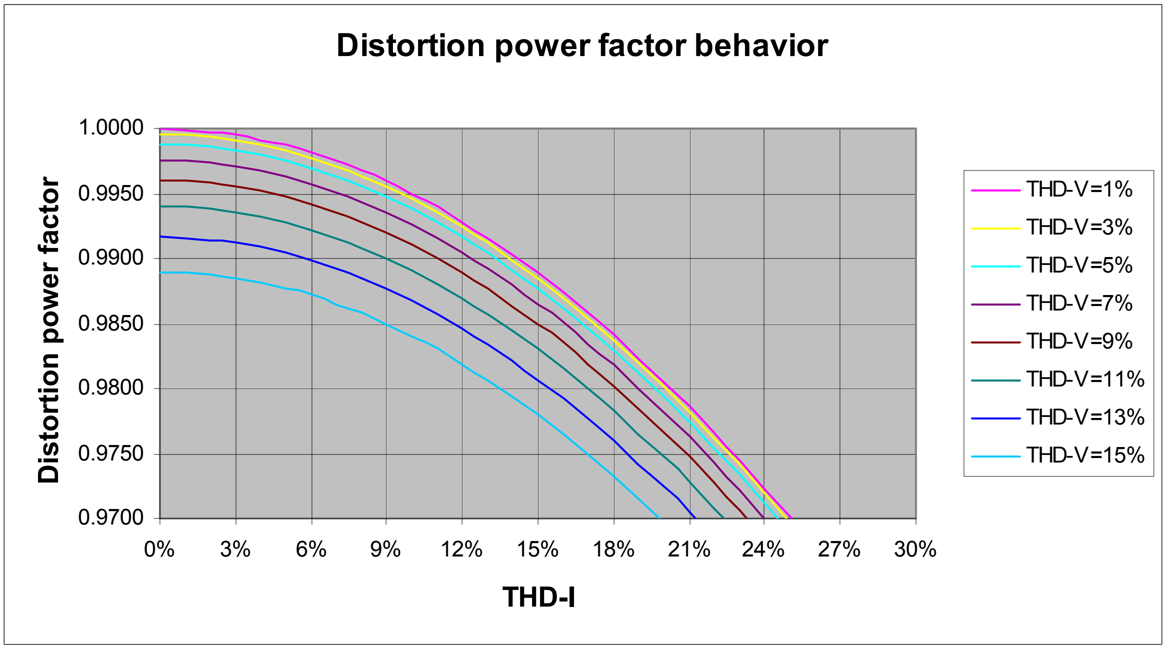

Figure 9

Example: Consider a load of 850 kW, 1000 kVA with 38% total harmonic current distortion. The displacement power factor is calculated as $850/1000 = 0.85$. The rms value of current is:

$$I = I_1\sqrt{1+0.38^2} \quad so \quad I = 1.0697 I_1$$

Therefore the total power factor is:

$$pf = \cos(\varphi_1)\frac{1}{\sqrt{1+THD_I^2}} = 0.85 \times \frac{1}{1.0697} = 0.795$$

Conclusion: While traditional methods of improving power factor may help to reduce some utility power factor charges, simple power factor correction does not assure adequate improvement of total power factor nor achieve the total benefits of performing total power factor correction. Only when measures are taken to correct both the displacement and distortion power factors can energy efficiency be achieved. The implementation of total power factor improvement can achieve significant cost savings such as elimination of utility power factor charges, reduced energy demand, reduced electrical equipment operating temperatures and extender electrical equipment life.